Study of Exotic Properties of Photonic Crystals

Niraj Kumar*
Department of Physics
Indian Institute of Technology, Kanpur

Mentor: Dr. R. Vijaya†
Professor, Department of Physics
Indian Institute of Technology, Kanpur
(Dated: November 29, 2013)

The project aims at studying the exotic properties of Photonic crystals. The tool employed to study the Photonic Band diagram calculations is the MEEP simlation software package developed at MIT. MEEP package implements finite-difference time-domain (FDTD) method for computational electromagnetism. This is a widely used technique in which space is divided into a discrete grid and then the fields are evolved in time using discrete time steps - as the grid and the time steps are made finer and finer, this becomes a closer and closer approximation for the true continuous equations, and one can simulate many practical problems essentially exactly.

In this report, the MEEP simulation pakage has been used to study different types of two-dimensional Waveguides, and to analyse the transmission , relflection and loss accossiated with the propagation of a Gaussian wavefront.

I. INTRODUCTION TO MEEP

MEEP(MIT Electromagnetic Equation Propagation) is the software simulation package developed at MIT to model the electromagnetic systems. It is based on a FDTD(finite-difference-time-domain) method. This works on the principle in which we divide the space into finite grids and the fields are then evolved in time using the discrete time units. Now, when we start to make the space and time grids finer and finer, this closely approximates to a continuous equations and simulation of Photonic crystals become feasible.

A. Maxwell's Equations

Meep package simulates the Maxwell's Equations. The equations for the evolution of fields are:

$$\frac{dB}{dt} = -\nabla \otimes E - J_{B} - \sigma_B B \; , B = \mu H \tag{1}$$

$$\frac{dD}{dt} = \nabla \otimes B - J - \sigma_D D \; , D = \epsilon E \tag{2}$$

Where **D** is the displacement field, ϵ is the dielectric constant, **J** is the current density (of electric charge), and **J** is the magnetic-charge current density. (Magnetic currents are a convenient computational fiction in some situations.) **B** is the magnetic flux density (often called the magnetic field), μ is the magnetic permeability, and **H** is the magnetic field. The σ_B and σ_D terms correspond to (frequency-independent) magnetic and electric conductivities, respectively.The divergence equations are implicitely:

$$\nabla \cdot B = -\int_0^t \nabla \cdot (J_B(t') + \sigma B)\, dt' \tag{3}$$

$$\nabla \cdot D = -\int_0^t \nabla \cdot (J_D(t') + \sigma B)\, dt' \equiv \rho \tag{4}$$

Most generally, ϵ depends not only on position but also on frequency (material dispersion) and on the field **E** itself (nonlinearity), and may include loss or gain

B. Units used in MEEP

In the above Maxwell's Equations, there are no constants like ϵ_0 , μ_0 and **c**. MEEP uses dimensionless units where all constants are unity. Maxwell's equations are scale invariant (multiplying the sizes of everything by 10 just divides the corresponding solution frequencies by 10), hence it is convenient in electromagnetic problems to choose scale-invariant units.
For this reason, the corresponding length scale is chosen a constant **a**. Moreover, since **c** = 1 in Meep units, **a** (or a/c) is our unit of time as well. In particular, the frequency **f** in Meep (corresponding to a time dependence e i2ft) is always specified in units of c/a (or equivalently ω is specified in units of 2πc/a), which is equivalent to specifying f as 1/T: the inverse of the optical period T in units of a/c. This, in turn, is equivalent to specifying f as a/λ where λ is the vacuum wavelength.

C. Finite-Difference-Time-Domain Method

FDTD methods divide space and time into a finite rectangular grid. If the grid has some spatial resolution Δx, then our discrete time-step Δt is given by Δt = SΔ x, where S is the Courant factor. In Meep, S = 0.5 by default (which is sufficient for 1 to 3 dimensions). This means that on doubling the grid resolution, the number of time steps doubles as well (for the same simulation

* http://kumarniraj.weebly.com
† http://www.iitk.ac.in/phy/New01/profile˙Vijaya.html

period). Thus, in three dimensions, on doubling the resolution, then the amount of memory increases by 8 and the amount of computational time increases by (atleast) 16.
In order to discretize the equations with second-order accuracy, FDTD methods store different field components at different grid locations. This discretization is known as a Yee lattice. As a consequence, Meep must interpolate the field components to a common point whenever we want to combine, compare, or output the field components. Although FDTD inherently uses discretized space and time, as much as possible Meep attempts to maintain the illusion that the user is using a continuous system. For instance, on specifying the dielectric function as a function $\epsilon(x)$ of continuous x, Meep is responsible for figuring out how they are to be represented on a discrete grid. If one wants to specify a point source, one simply specifies the point x where one want the source to reside- Meep will figure out the closest grid points to x and add currents to those points, weighted according to their distance from x.

II. STRAIGHT WAVEGUIDE

A computational cell is defined prior to defining the stucture of the waveguide. A computational cell of size 16 × 8 is created and a horizontal waveguide of ϵ (non dispersive) 12 is created at the center.

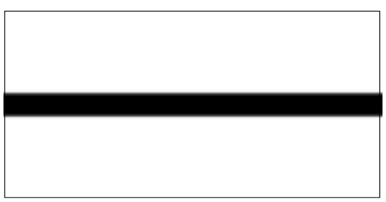

FIG. 1. Straight Waveguide (Black = Waveguide section)

A continuous point current J_z source of frequency 0.15 is turned on at t=0. Thus, 0.15 corresponds to a vacuum wavelength of about 1/0.15 = 6.67, or a wavelength of $6.67/\sqrt{11} \approx 2$ in the $\epsilon = 12$ material. Thus, the waveguide is half a wavelength wide. The current is located at (7,0), which is 1 unit to the right of the left edge of the cellthis is done to keep the boundary conditions from interfering with them.
The boundary used here is the Perfectly Mathched Layer(PML) of thickness 1 which is spread around the waveguide. It absorbs the radiating fields going outside the Waveguide.
The resolution is set to 10, which corresponds to around 67 pixels/wavelength, or around 20 pixels/wavelength in the high-dielectric material. Also, the unit cell is 160 × 80. By the variable *resolution*, the number of pixels per distance unit is specified.
Figures 2-7 below depict the propagation of E_Z with diffrent Meep time units:

FIG. 2. E. field propagation at t=0 units

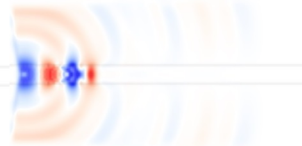

FIG. 3. E. field propagation at t=22 units

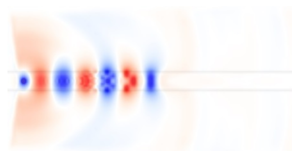

FIG. 4. E. field propagation at t=42 units

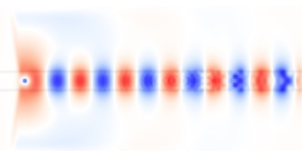

FIG. 5. E. field propagation at t=107 units

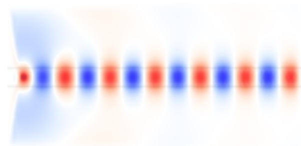

FIG. 6. E. field propagation at t=237 units

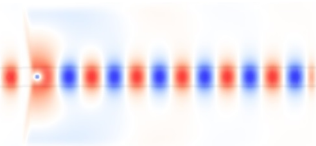

FIG. 7. E. field propagation at t=329 units

III. 90° BEND WAVEGUIDE

A computation cell of size 16 × 16 is created and a 90° Bend Waveguide of $\epsilon = 12$ is created inside the computational cell. The horizontal and vetical lenghths of the waveguide are 12 units. The centre of horizontal waveguide is (-2,-3.5) and the centre of the vertical waveguide being (3.5,2). This can be seen in the Figure 8 of the waveguide below.

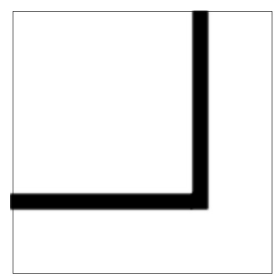

FIG. 8. 90 Bend Waveguide (Black = Waveguide section)

A Gaussian current J_z source of cental frequency 0.15 and frequency width 0.10 is turned on at t=0. A point source does not couple very efficiently to the waveguide mode, so we'll expand this into a line source the same width as the waveguide by adding a *size* property to the source. And, effectively, instead of turning on current source abruptly(this results in the formation of speckels in the field pattern) at t=0, the pulse is turned on slowly because of the *size* property of the source. The current is located at (7,-3.5)(bottom horizontal part), which is 1 unit to the right of the left edge of the cellthis is done to keep the boundary conditions from interfering with them.
The boundary used here again is the Perfectly Mathched Layer(PML) of thickness 1 which is spread around the waveguide. It absorbs the radiating fields going outside the Waveguide.

The resolution is set to 10, which corresponds to around 67 pixels/wavelength, or around 20 pixels/wavelength in the high-dielectric material. Also, the unit cell is 160 $\times$ 160. By the variable *resolution*, the number of pixels per distance unit is specified. Figure 9-14 give the snapshots of propagation of E_Z at t Meep time units:

FIG. 9. E. field propagation at t=0 units

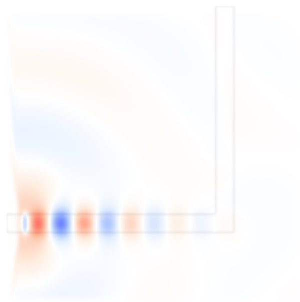

FIG. 10. E. field propagation at t=112 units

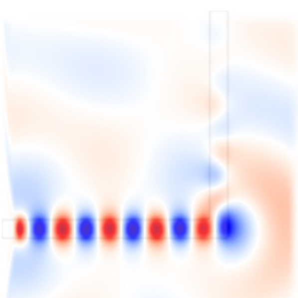

FIG. 11. E. field propagation at t=177 units

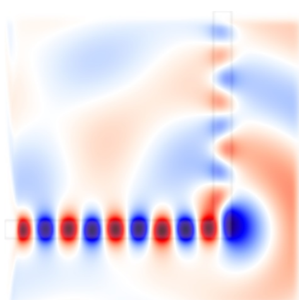

FIG. 12. E. field propagation at t=247 units

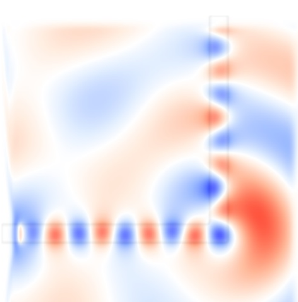

FIG. 13. E. field propagation at t=302 units

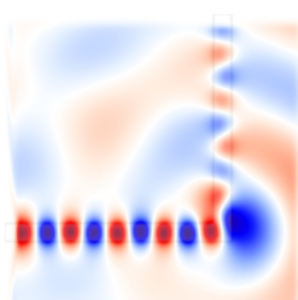

FIG. 14. E. field propagation at t=329 units

Now in order to calculate the transmission and reflection spectrum, the location is needed to specified where these flux spectra needs to be calculated and at what frequencies these need to be calculated. The guassian spread of frequencies is divided into 100 parts and the fluxes are caluclated for these 100 different frequencies cetred at f=0.25 and ranging from 0.10 to 0.20.
While computing the Tranmisson spectum, the flux is calculated at the top vertical portion (3.5,7). Since in Meep, we are using scale invariant units, we have to normalize this tranmission flux, so this is divided by the equivalent flux that would be found in a straight waveguide of same width inside the computational cell. This gives us the Transmission specturm.
Meep employes the technique of calculating the reflection spectrum whereby the fourier transformed normalized fields calculated during the computation with straight waveguide is loaded and subtracted before the other simulation runs. The reflection spectrum is calculated at the coordinate (-7,-3.5) for 100 different frequency values. Figure 15 shows the plot of transmission, reflection and loss associated with the propagation of E field in the waveguide.
The loss associated with the porpagation is calculated by using Loss = 1 - Transmission - Reflection.
Figure 15 is a plot of Transmission, Reflection and Loss during the propagation for 100 values of frequency.

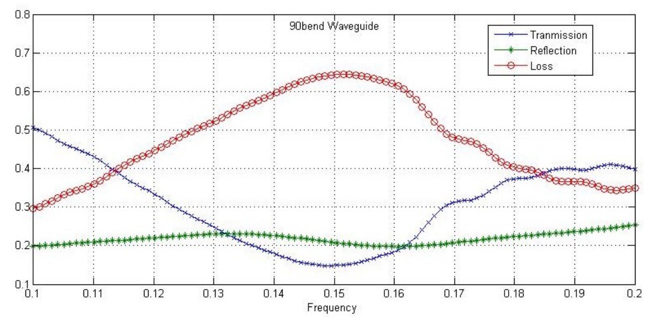

FIG. 15. Transmission,Reflection and Loss in T-Waveguide)

IV. T - SHAPE WAVEGUIDE

A computation cell of size 16 $\times$ 16 is created and a T-shape Bend Waveguide of $\epsilon = 12$ is created inside the computational cell. The horizontal lenghths of the waveguide are 12 units. The centre of horizontal waveguide is (-2,0), while for the vertical waveguide, it is (3.5,0). This can be seen in the Figure 16 of the waveguide below.

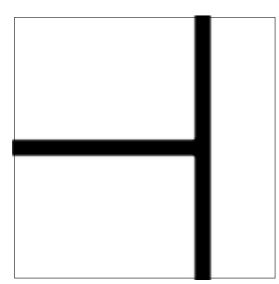

FIG. 16. T- shape Waveguide (Black = Waveguide section)

Same Guassian current source was employed inside the waveguide and Figures 17-22 depict the propagation of E_Z at t Meep time units:
The transmission, reflection and loss calculation of fluxes is done analogous to the one in 90° Bend Waveguide. The Guassian current source is located at (-7,0) , and the transmission spectrum are calculated at (3.5,7) and (3.5,-7) , while reflection spectrum are

FIG. 17. E. field propagation at t=0 units

FIG. 18. E. field propagation at t=82 units

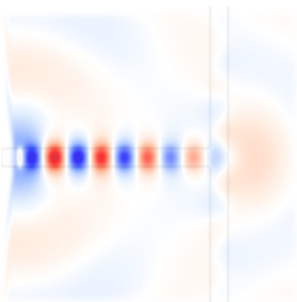

FIG. 19. E. field propagation at t=152 units

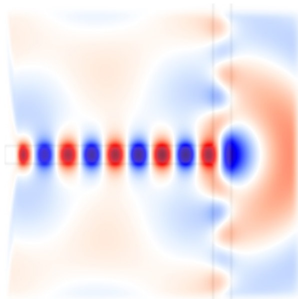

FIG. 20. E. field propagation at t=212 units

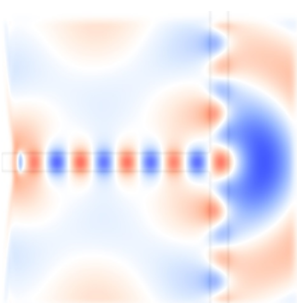

FIG. 21. E. field propagation at t=262 units

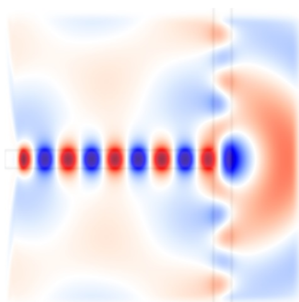

FIG. 22. E. field propagation at t=327 units

calculated at (-7,0).

Figure 23 is a plot of Transmission, Reflection and Loss during the propagation for 100 values of frequency.

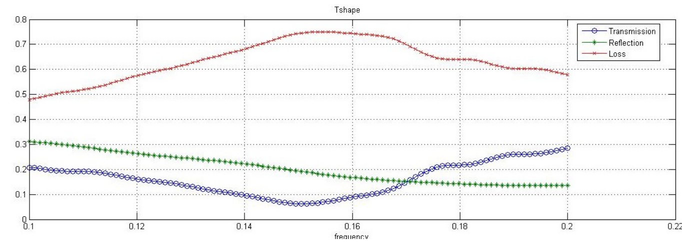

FIG. 23. Transmission,Reflection and Loss in T-Waveguide)

V. CONCLUSION

The primary focus of this first half of the project was on understandiing the implementation of Meep by considering the three different types of waveguides and looking at the fluxes emerging because of the current source propagation through the waveguide. The refelction, transmission and loss assciated with the propagation were calculated and plotted for different frequencies in the frequency spectrum of Guassian wavefront. The losses were accounted because the PML boundary layers which forced the fields to go to 0 immediately at the boundary.

The next goal is understanding and implementing the Meep pacakge for various different 3-d crystals with different boundary conditions(periodic boundary condition) . The focus would be on calcuation of trasmission, reflection and losses assciated with the propagation of E-field inside the crystal and to calculate the resonant modes.

[1] Photonic Crystals, Modling the flow of Light, John D. Joannopoulos, Steven G. Johnson, Joshua N. Winn, and Robert D. Meade 2nd Edition, 2008.
[2] MEEP Ab-Initio, Developed at MIT http://ab-initio.mit.edu/wiki/index.php/meep
[3] Meep: A flexible free-software package for electromagnetic simulations by the FDTD method Ardavan F. Oskooia, Corresponding author contact information, David Roundyb, Mihai Ibanescua, Peter Bermel, J.D. Joannopoulosa, Steven G. Johnsona Elsevier, Volume 181, Issue 3, March 2010, Pages 687702